William L. Blubaugh and Kristin Emmons

ALGEBRA FOR ALL

Graphing for All Students

TEACHER'S GUIDE

For years, many of us have known that an understanding of concepts from algebra can be a prerequisite for many careers. Changes in society and expectations of employers have made algebraic ideas even more necessary than in the past, and more people are recognizing the significance of algebraic thinking. Over the past ten years, many documents have indicated the need to provide algebra experiences to all students (NCTM 1989; Glatzer and Choate 1992). As a result, high schools across the country are changing graduation requirements to include or expand the study of algebra. This step in the right direction creates the challenge of offering an algebra curriculum that meets the needs of all students.

One challenge in furnishing a curriculum for all students is that some students have stronger background skills than others. Although some students come into an algebra classroom with a strong sense of numbers and number operations, others are struggling with these ideas. One way to bridge, or narrow, this difference is to present intuitive experiences in a prealgebra course or at the beginning of the algebra course.

For example, the concepts of variable and function are fundamental to the study of algebra. In learning these concepts, students need to understand and work with symbolic, graphical, tabular, and verbal representational forms. Many teachers are already including pattern activities in which students generate tables and rules to fit a situation, such as the "handshake problem"; paper-folding; pattern-block trains; perimeter and area extensions; and more. Often what is lacking is a visual picture of the relationship that goes beyond tables and formulas.

A particularly challenging area for prealgebra students involves interpreting graphs and charts. This article presents activities that can be used in a small-group, student-centered approach. Within these activities students are expected to—

- explain their choice of a particular graph and describe how they would label the axes,
- write stories that "fit" a given graph, and
- produce graphs that describe real-world situations.

Much of the graphing that is done in classrooms involves simply plotting points that have already been given in tabular form. The real-world relationship between the variables—and hence the function concept—is often lost. Students would be better able to make these connections through experiences that are more intuitive and less contrived.

The activities in this article support students as they develop their intuitive notions of graphs. Students should be able to describe the relationships expressed in the graphs and to make generalizations of inferences on the basis of the visual image. Throughout the activities, the independent variable represents time, so that students can focus on the dependent variable.

Often what is lacking is a visual picture of the relationship

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This section is designed to provide in reproducible formats mathematics activities appropriate for students in grades 7–12. This material may be reproduced by classroom teachers for use in their own classes. Readers who have developed successful classroom activities are encouraged to submit manuscripts, in a format similar to the "Activities" already published, to the senior journal editor for review. Of particular interest are activities focusing on the Council's curriculum standards, its expanded concept of basic skills, problem solving and applications, and the uses of calculators and computers.

Write to NCTM, attention: infocentral, or send e-mail to infocentral@nctm.org, for the catalog of educational materials, which lists compilations of "Activities" in bound form.—Ed.

Sheet 1

Some leading questions may help the class or individual students who do not seem to know where to start. If necessary, the teacher can begin by asking such questions as these:

1. What does the height of the graph tell you?
2. What is happening to the time as we move to the right?
3. What does the horizontal line in the graph labeled "George's Dad" mean?
4. Why does George's graph touch the horizontal axis at a point closer to the vertical axis than the graph for George's dad?
5. How would you have to be eating to make the graph be a horizontal line?
6. What possible reasons prevent George's mom's graph from touching the vertical axis?
7. What is the significance of the x- and y-intercepts?

Students should then share their interpretations of the graph with the class. After reaching some agreement on these ideas, students begin the next activity.

Sheet 2

The first part of this activity asks students to construct a pair of graphs to represent George's popcorn-eating pattern and that of his friend Alyssa. Students need to consider in their graphs that George and Alyssa both finish eating at the same time. The second part of this sheet asks students to think about and construct a graph that represents their own eating habits.

Students should be able to make intelligent inferences from graphs

Sheet 3

This sheet asks students to explore the meaning of a graph as the relationship between the independent variable, on the horizontal axis, and the dependent variable, on the vertical axis. Possible solutions are the following: 1-A, 2-G, 3-C, 4-F, 5-E, 6-B, and 7-D. Some other matching might make sense as well.

Sheet 4

If students have difficulty in thinking of a possible situation depicted by the two graphs, the teacher might make the following suggestions. In question 1, students can consider the speed of a car that is traveling on a short side street with stop signs at each end of the street. In question 2, students can consider a student walking home from school. The student starts walking at a constant speed, then slows down and stops for a while to talk with a friend, then starts walking again at a faster yet constant pace until arriving home. The pace is faster after the stop, possibly to arrive at home on time. The students should label each axis. The teacher may wish to continue with such graphs until the students understand the increasing and decreasing nature of a function graphically.

Sheet 5

These four activities have students continue developing reasonable graphs. Again, students should label both axes appropriately and be prepared to explain their reasoning to their classmates.

In summary, we have presented a few examples of graphing explorations that can assist students during the early part of their algebra experiences. The teacher can expand these graphing ideas throughout the school year and relate them to the students' common experiences. Doing bouncing-ball activities or experimenting with Newton's laws of motion can incorporate more graphs. Such activities can improve students' interpretations of graphs. Involving students with pattern activities to generate "function" rules allows students to experience the solutions graphically.

After students have had experiences communicating through general graphs, they will be much better prepared to make intelligent inferences from graphs. Students will be better able to interpret graphs and understand the relationship between the variables that the graph illustrates. The activities presented in this article should help all students build a conceptual basis for the graphs that they will use in other courses, help them interpret real-world graphs, and prepare them for more formal algebraic experiences.

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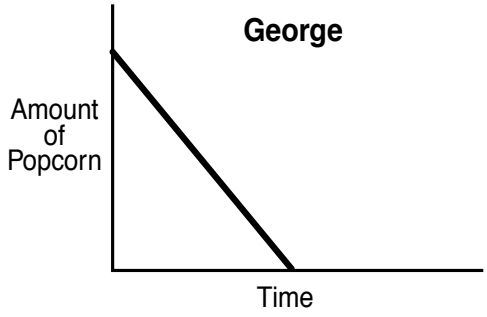
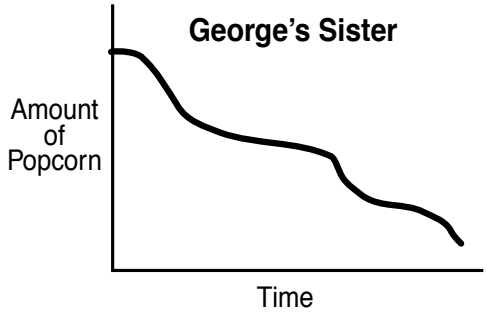
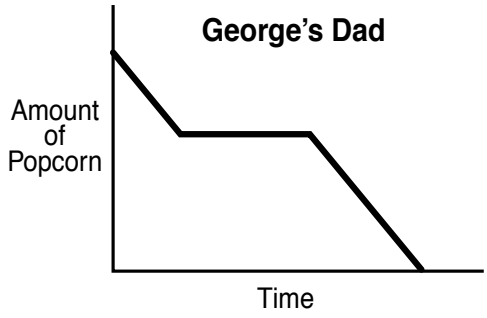
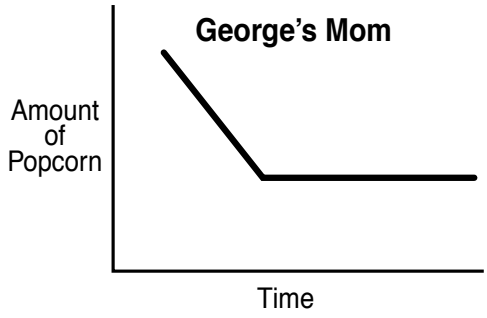
(Worksheets begin on page 325)

George and his family were watching a movie and eating popcorn. Each family member had a bowl with the same amount of popcorn. The graphs below all show the amount of popcorn remaining in the person's bowl over a period of time. Under each graph, write a few sentences describing what may have happened.









From the *Mathematics Teacher,* April 1999

George and his friend Alyssa went to the movies, where they each bought a medium tub of popcorn. George was quite hungry and quickly ate half his popcorn, paused for a moment, and then continued eating at the same rate as before. Alyssa waited for a few minutes before she began and then ate at a steady rate. When she noticed that George had almost finished his popcorn, she gave him some of her own. They then both continued eating and finished at the same time.

1. On the axes below, show how the amount of popcorn remaining in George's tub varied over time.
2. On the same axes but using a different color, show how the amount of popcorn remaining in Alyssa's tub varied over time.
3. Examine each graph. How are the two graphs similar, and how they are different?

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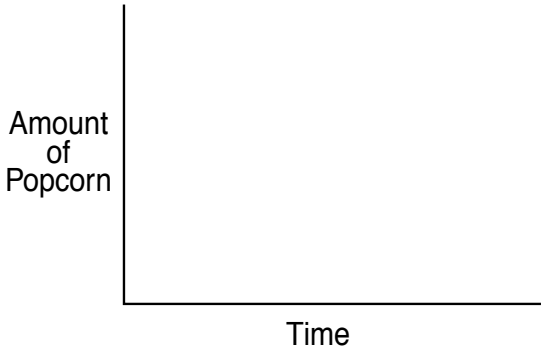
_____
_____



4. How does your graph show that Alyssa waited for a few minutes before she began eating?

_____

5. How did you show on your graph that she gave him some of her own popcorn?

_____
_____

6. How does your graph show that they both continued eating and finished at the same time?

_____
_____

7. How does your hunger change from morning until night? On the given axes, show how your hunger changes from the time you wake up in the morning until you go to bed at night.
8. Write a paragraph explaining what your graph reveals about your eating habits.

_____
_____
_____
_____
_____
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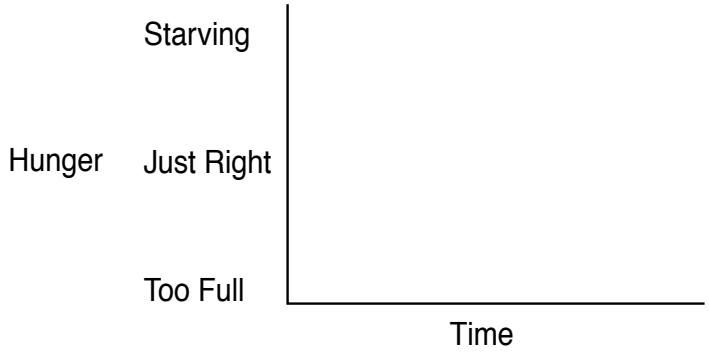


(Continued on page 332)

(Continued from page 326)

WHAT IS HAPPENING? SHEET 3

Match each of the following seven scenarios with the most appropriate graph given. As you look at each graph from left to right, remember that time is advancing.

___ 1. We rode the roller coaster steadily to the top, then went faster and faster as we went down the other side. The speed of the roller coaster is the dependent variable of the graph, that is, the variable on the vertical axis.

___ 2. The kettle heats before the corn begins to pop. The corn starts to pop and continues popping until almost all the corn has popped. The amount of *un*popped corn in the kettle is the dependent variable.

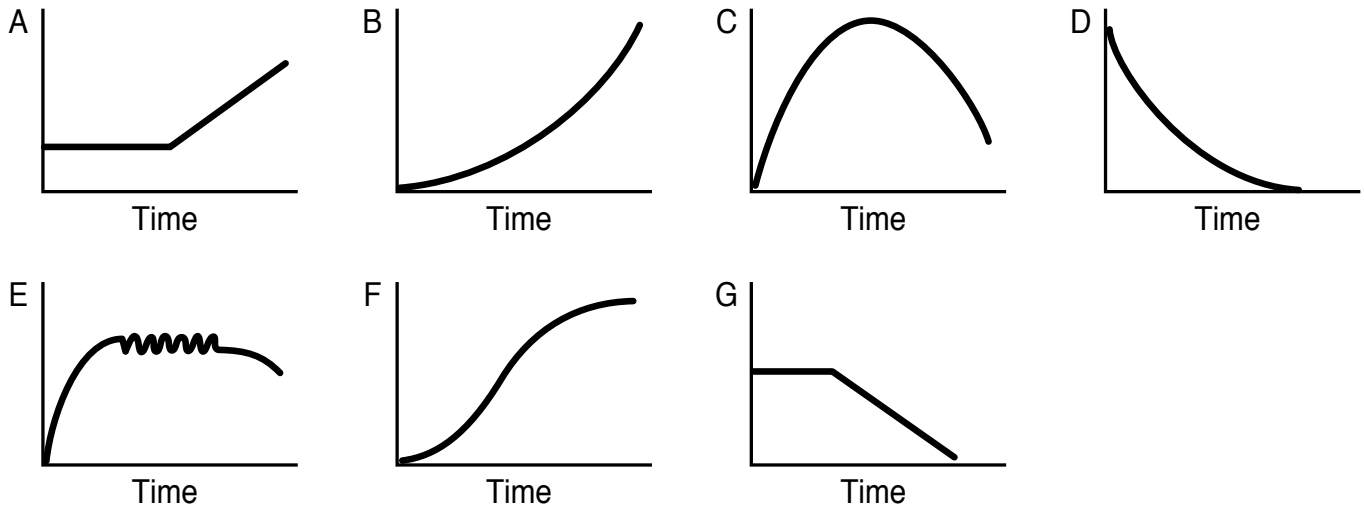
___ 3. A balloon was blown up in class and then let go. It flew around the room. The amount of air in the balloon is the dependent variable.

___ 4. At the beginning of spring, the grass grew slowly and I seldom had to mow the lawn. By midsummer it was really growing, so I mowed twice a week. In fall, I only mow once in a while. The number of lawn mowings to date is the dependent variable.

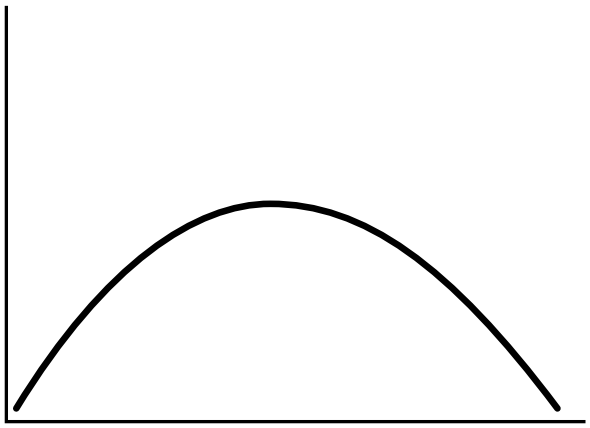
___ 5. I turned the oven on. When it was hot, I put in the cake. The cake baked for about thirty minutes. I turned the oven off and removed the cake. The oven temperature is the dependent variable.

___ 6. We bought a pair of rabbits last year. They have had several litters, and we have so many rabbits that the pens are full. If more are born, we will have to give some away or find room for the new ones. The number of rabbits is the dependent variable.

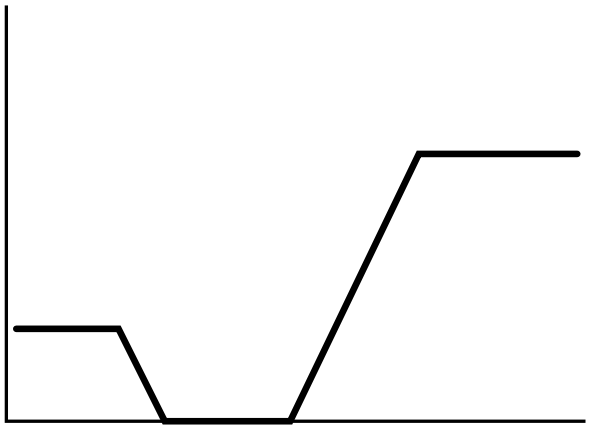
___ 7. I put water in the ice-cube tray and placed it in the freezer. The temperature of the water in the ice-cube tray is the dependent variable.



1. Think of a real-life situation that could be represented by the graph on the right. Write a story about the situation, and be prepared to read your story to the class. Be sure to label the axes of the graph.



2. Think of a real-life situation that could be represented by the graph on the right. Write a story about it, and be prepared to read the story to your classmates. Be sure to label the axes of the graph.



Before graphing each relationship, label both axes appropriately, and be ready to explain to your classmates the reasoning behind each of your graphs.

1. You turn on the hot-water faucet. The temperature of the running water depends on the number of seconds since you turned on the faucet.

2. As you play with a yo-yo, the yo-yo's distance from the floor depends on the number of seconds that have passed since you started.

1.

2.

3. You go from sunlight into a dark room. The diameter of your pupils depends on the length of time that you have been in the room.

4. You pour some cold water from the refrigerator into a glass and leave it on the counter. As the glass sits on the counter, the water's temperature depends on the number of minutes that have passed since you poured it.

3.

4.